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Characterization of Nonsymmetric Forms of Fully Orthotropic Laminates

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Stacking-sequence listings are presented in abridged form for fully orthotropic angle-ply laminates with up to 21 plies, which are characterized in terms of angle- and cross-ply subsequence symmetries. The abridged set of sequences is derived from a new definitive list that supersedes previously published listings. Laminate stacking sequences are presented together with dimensionless parameters from which the extensional and bending stiffness terms are readily calculated and an assessment of the bending stiffness efficiency can be made for angle- and cross-ply subsequences. Expressions relating the dimensionless parameters to the well-known lamination parameters are also given, together with graphical representations of feasible domains for all subsequence symmetries contained in the definitive list. Feasible domains for extensionally isotropic and fully isotropic laminates are also presented as important subsets of fully orthotropic laminates. Finally, examples are given for tapered laminates with fully orthotropic properties, derived from compatible sequences in the definitive list.

Nomenclature

$\mathbf{A}$, A_{ij}	=	extensional (membrane) stiffness matrix and its elements (i, $j = 1, 2, 6$)
$\mathbf{B}$, B_{ij}	=	bending-extension-coupling stiffness matrix and its elements (i, $j = 1, 2, 6$)
$\mathbf{D}$, D_{ij}	=	bending (flexural) stiffness matrix and its elements (i, $j = 1, 2, 6$)
H	=	laminate thickness ($n \times t$)
n	=	number of plies in the laminate stacking sequence
Q_{ij}	=	reduced stiffness (i, $j = 1, 2, 6$)
Q'_{ij}	=	transformed reduced stiffness (i, $j = 1, 2, 6$)
t	=	ply thickness
ζ, $\zeta_{\pm}$, $\zeta_{\circ}$, $\zeta_{\bullet}$	=	bending stiffness parameter for laminate, angle-ply, and cross-ply subsequences
ξ_1^A, ξ_2^A	=	lamination parameters for extensional stiffness
ξ_1^D, ξ_2^D	=	lamination parameters for bending stiffness ($\xi_1^D = \xi_9$, $\xi_2^D = \xi_{10}$)
$+$, $-$, $\pm$	=	angle plies, used in the stacking-sequence definition
○, ●	=	cross plies, used in the stacking-sequence definition

Subscripts

F	=	elements all finite
I	=	isotropic form [see Eqs. (3–5)]
S	=	specially orthotropic form [see Eqs. (1) and (2)]
0	=	elements all zero

I. Introduction

COMPOSITE laminate materials are typically characterized in terms of their response to mechanical or thermal loading, which is generally associated with a description of the coupling behavior, unique to this type of material [i.e., coupling between in-plane (extension or membrane) and out-of-plane (bending or flexure) actions, coupling between in-plane shear and extension, and coupling between out-of-plane bending and twisting]. One such classification system is offered by the Engineering Sciences Data Unit (ESDU) [1], in which the extensional $\mathbf{A}$, coupling $\mathbf{B}$, and bending $\mathbf{D}$ stiffness matrices are used together with an extended subscript notation to describe the form of the elements in each matrix. For instance, balanced and symmetric stacking sequences, which generally possess bending anisotropy, are referred to by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$, signifying that the elements of the extensional stiffness matrix $\mathbf{A}$ are specially orthotropic in nature; that is,

$$A_{16} = A_{26} = 0 \tag{1}$$

the bending-extension coupling matrix $\mathbf{B}$ is null, and all elements of the bending stiffness matrix $\mathbf{D}$ are finite.

Laminates possessing extensional anisotropy give rise to coupling between in-plane shear and extension only and, by the same rationale, are referred to by the designation $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$, signifying that all elements of the extensional stiffness matrix $\mathbf{A}$ are finite, the bending-extension coupling matrix $\mathbf{B}$ is null, and the elements of the bending stiffness matrix $\mathbf{D}$ are specially orthotropic in nature; that is,

$$D_{16} = D_{26} = 0 \tag{2}$$

A designation for extensional anisotropy is not listed as part of the 10 laminate classifications described in [1], but is, however, the subject of a recent paper [2] identifying the definite list of $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ stacking sequences with up to 21 plies, thus complementing a new definitive list [3] of fully orthotropic laminates (FOLs). Note that the term FOL is synonymous with specially orthotropic laminates, which possess none of the coupling characteristics described previously and are represented by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$.

A related paper [4] presents the characterization of extensionally isotropic laminates (EILs), with the designation $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_S$, and fully isotropic laminates (FILs), with the designation $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$. These laminates represent a subset of FOLs and are contained within the definitive list [3], because in addition to the specially orthotropic form of each matrix [see Eqs. (1) and (2)], elements simplify further in EILs, where the designation $\mathbf{A}_S$ is replaced with $\mathbf{A}_I$ to indicate that

$$A_{11} = A_{22} \tag{3}$$

and

$$A_{66} = (A_{11} - A_{12})/2 \tag{4}$$

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and further still in FILs, where the designation $\mathbf{D}_S$ is replaced with $\mathbf{D}_I$ to indicate that

$$D_{ij} = A_{ij}H^2/12 \tag{5}$$

where H is the laminate thickness corresponding to the total number of plies n of thickness t.

Fully orthotropic laminates minimize distortion during manufacturing and maximize compression buckling strength [5] in comparison with balanced and symmetric laminates, which generally possess flexural anisotropy and are therefore of continuing interest to industry and the academic community alike. For instance, Valot and Vannucci [6] provided examples of FOLs with antisymmetric sequences, following a previous paper [7] on FILs. Indeed, these two papers are part of, and provide reference to, a growing number of related papers by a community of coworkers addressing the development of laminate stacking sequences for a range of behavioral characteristics, which can be traced back to an original paper by Caprino and Crivelli-Visconti [8], identifying the specially orthotropic angle-ply laminate with eight plies. Much of this related work is focused on establishing stacking sequences for EILs, FILs, and laminates with material homogeneity. However, with the exception of FILs, these material characterizations generally provide little distinction between laminates with different coupling behavior. For instance, the quasi-homogenous laminate, which is defined as possessing the same elastic properties in both flexural and membrane actions, satisfying Eq. (5), but with no coupling between these two actions, may be designated as either $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_F$ or $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$, and EILs may be designated as either $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_F$ or $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_S$. Such sequences have been obtained through an inverse polar (or Mohr's circle) representation method developed by Verchery [9], but acknowledged [10] to be similar to the method presented previously by Tsai and Pagano [11].

In contrast, original work by Bartholomew [12], which forms the basis of the Engineering Sciences Data Unit (ESDU) publication [13] for the so-called definitive list of fully orthotropic angle-ply laminates, precedes the findings of Caprino and Crivelli-Visconti [8], but appears to have been completely overlooked in the literature described previously. Reference [13] contains 75 symmetric sequences for laminates with up to 21 plies and 653 antisymmetric sequences for laminates with up to 20 plies, together with 49 additional nonsymmetric (asymmetric) sequences that were derived by combining the symmetric and antisymmetric sequences. Further inspection reveals that there are no angle-ply laminates possessing specially orthotropic properties with fewer than 7 plies. Indeed, there is but a single generic 7-layer angle-ply antisymmetric stacking sequence. This number increases to 233 generic antisymmetric sequences with 20-ply layers. There are no symmetric stacking sequences with less than 12 layers and only 25 generic combinations with 20 layers. These 25 generic stacking sequences possess balanced and symmetric combinations of angle plies, together with cross plies, which may be 0 and/or 90 deg, symmetrically disposed about the laminate midplane; all possess angle-ply layers on the outer surfaces of the laminate. The term *generic* is used here to describe the form of the stacking sequences adopted, defined by three parameters: $+$, $-$, and $*$, relating to positive and negative angle plies with general orientation, θ, and cross ply, respectively.

The derivation [12] adopted in [13] made the explicit assumption that cross plies, as well as angle plies, are symmetrically disposed about the laminate midplane; that is, the mixing of 0 and 90 deg plies is permitted only in one-half of the laminate, which is then reflected symmetrically about the laminate midplane. This rule applies to both symmetric and antisymmetric angle-ply stacking sequences. For this reason, cross plies are legitimately represented by the single parameter $*$.

The relatively small number of fully orthotropic sequences for thin laminates clearly leaves limited scope for composite tailoring, particularly when ply terminations are necessary and fully orthotropic characteristics are a design requirement. This was the key motivation leading to the redevelopment of a definitive list [3] for specially orthotropic angle-ply laminates with up to 21 plies, presented in the current paper in abridged form.

In the derivation of this list for (but not restricted to) standard angle-ply configurations (i.e., ± 45, 0 and 90 deg), the general rule of symmetry is relaxed. Cross plies, as well as angle plies, are therefore no longer constrained to be symmetric about the laminate midplane, leading to an increase in the number of possible solutions. For 16-ply laminates, there are approximately one billion (10^9) possible stacking-sequence combinations, of which 360 comply with the requirements of special orthotropy, increasing to approximately one trillion (10^{12}) combinations for 21 plies, giving rise to a hundredfold increase in the number of fully orthotropic laminates. The scope for composite tailoring is now vastly increased, particularly in the context of fully orthotropic tapered laminates. The numbers of sequences for each ply-number grouping are summarized in Table 1, which also provides cross-referencing to the tables of abridged listings that follow.

Because of the substantial number of sequences identified in the definitive list, it is beneficial for design purposes to express the stiffness properties in terms of lamination parameters, which can be conveniently presented in graphical form, as originally conceived by Fukunaga and Vanderplaats [14] for the purposes of optimum design: the membrane or flexural stiffness terms are now defined by two linear design variables. Optimized lamination parameters may then be matched against a corresponding set of laminate stacking sequences. Graphical representations of feasible domains of lamination parameters are readily extended to other laminate classifications: EILs are reduced to a single-line representation, and FILs are represented by a single point.

Table 1 Number of FOLs for 7–21-ply laminates corresponding to prefix designations for antisymmetric A, cross-symmetric C, nonsymmetric N, and symmetric S ply subsequences, listed in abridged form in Tables 2–9

	Number of plies, n[a]															
Prefix[b]	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Table
AC	——	——	——	——	——	——	——	——	24	12	84	——	——	90	——	2
AN	——	——	——	——	——	——	——	——	24	——	232	132	1984	920	11,240	3
AS (A)	2 (1)	1 (1)	6 (2)	6 (3)	24 (4)	21 (6)	84 (10)	76 (14)	288 (26)	268 (34)	1002 (65)	934 (89)	3512 (165)	3290 (233)	12,392 (–)	4
$_{+}NS_{+}$	——	——	——	——	——	——	——	——	——	2	12	8	64	94	40	5
$_{+}NS_{-}$	——	——	——	——	——	——	——	——	——	——	——	16	24	88	168	6
$_{+}NN_{+}$	——	——	——	——	——	——	——	——	4	4	136	120	476	788	3,970	7
$_{+}NN_{-}$	——	——	——	——	——	——	——	——	——	24	116	124	1444	1,372	12,108	8
$_{+}NN_{\bigcirc}$	——	——	——	——	——	——	——	——	16	15	188	151	878	838	7955	9
SC	——	——	——	——	——	——	——	——	——	2	4	——	——	6	——	10
SN	——	——	——	——	——	——	——	——	——	——	8	8	64	88	40	11
SS (S)	——	——	——	——	——	4 (1)	——	12 (3)	4 (1)	33 (6)	50 (4)	110 (13)	120 (6)	352 (25)	344 (16)	12

[a]Numbers given in parentheses represent those listed in [13].
[b]Subscripts arranged before and after the prefix designations denote angle ($+, -$) and cross plies ($\bigcirc = 0$ or 90 deg) and correspond to the orientations of the top and bottom plies of the laminate, respectively.

II. Stacking-Sequence Derivation

In the derivation of the stacking sequences that follow, the general rule of symmetry is relaxed. Cross plies, as well as angle plies, are therefore no longer constrained to be symmetric about the laminate midplane. Consequently, the mixing of 0 and 90 deg plies needs special attention to avoid violation of the rules for special orthotropy [see Eqs. (1) and (2)]. Examples of symmetric angle-ply laminates with nonsymmetric cross-ply subsequences have been presented previously [15] in the context of composite plate instability and represent one of the many nonsymmetric forms contained in the definitive list of FOLs possessing standard angle-ply configurations (e.g., ±45, 0 and 90 deg), many of which appear both extraordinary in appearance and infeasible in terms of the uncoupled behavior that the laminates possess. The great majority of FOLs are nonsymmetric and many of these are without any subsequence patterns (e.g., symmetries or repeating groups), which is contrary to the assumptions on which many previous studies have been based.

For compatibility with the previously published data, similar symbols have been adopted for defining all stacking sequences that follow. Additional symbols and parameters are necessarily included to differentiate between cross plies (0 and 90 deg), given that symmetry about the laminate midplane is no longer assumed.

The resulting sequences are characterized by subsequence symmetries using a double-prefix notation; the first character relates to the form of the angle-ply subsequence, and the second character relates to the cross-ply subsequence. The double prefix contains combinations of the following characters: *A* for antisymmetric form, *N* for nonsymmetric, and *S* for symmetric. Additionally, for the cross-ply subsequence only, *C* is used to indicate cross-symmetric form.

To avoid the trivial solution of a stacking sequences with cross plies only, all sequences have an angle-ply (+) on one outer surface of the laminate. As a result, the other outer surface may have an angle-ply of equal (+) or opposite (−) orientation or a cross ply (○), which may be either 0 or 90 deg. A subscript notation using these three symbols is employed to differentiate between similar forms of sequence.

The form (and number) of the stacking sequences contained in the definitive list [3] for up to 21 plies can therefore be summarized as *AC* (210), *AN* (14,532), *AS* (21,906), *SC* (12), *SN* (192), *SS* (1029), ${}_{+}NS_{+}$ (220), ${}_{+}NS_{-}$ (296), ${}_{+}NN_{+}$ (5498), ${}_{+}NN_{-}$ (15,188), and ${}_{+}NN_{\bigcirc} = {}_{+}NN_{\bullet}$ (10,041).

As adopted in the published ESDU listings [13], the new sequences are ordered in terms of ascending numbers of plies n or $\zeta(=n^3)$, which are in turn ordered by ascending value of the bending stiffness parameter for the angle plies ($\zeta_{\pm}$) and then by one of the two cross-ply subsequences $\zeta_{\bigcirc}$ within the laminate. Hence, the numbering of sequences for each subsymmetric form, described in the previous section, may be readily extended.

The calculation of the bending stiffness parameter $\zeta_{\pm}$ is readily demonstrated for the 7-ply laminate, designated as *A*381 in the ESDU listings [13] and as *AS*2 in the definitive list [3], with stacking-sequence $[\pm/-/\bigcirc/+/\pm]_T$, where the bending stiffness terms

$$D_{ij} = \sum_{k=1}^{n} Q'_{ij}(z_k^3 - z_{k-1}^3)/3 \tag{6}$$

may be written in sequence for the 7 individual plies, where z, representing the distance from the laminate midplane, is expressed here in terms of the uniform ply thickness t:

$$\begin{aligned} D_{ij} = \{&Q'_{ij+}((-5t/2)^3 - (-7t/2)^3) + Q'_{ij-}((-3t/2)^3 \\ &- (-5t/2)^3) + Q'_{ij-}((-t/2)^3 - (-3t/2)^3) + Q'_{ij\bigcirc}((t/2)^3 \\ &- (-t/2)^3) + Q'_{ij+}((3t/2)^3 - (t/2)^3) + Q'_{ij+}((5t/2)^3 \\ &- (3t/2)^3) + Q'_{ij-}((7t/2)^3 - (5t/2)^3)\}/3 \end{aligned} \tag{7}$$

where subscripts $i, j = 1, 2, 6$.

The bending stiffness contribution from the angle plies is therefore

$$D_{ij\pm}(=D_{ij+} + D_{ij-}) = 85.5t^3/3 \times Q'_{ij\pm} \tag{8}$$

and for orthotropic plies,

$$D_{ij_{\bigcirc}} = 0.25t^3/3 \times Q'_{ij_{\bigcirc}} \tag{9}$$

Note that the decoupling of in- and out-of-plane actions for the laminate may be verified in a similar manner from

$$B_{ij} = \sum_{k=1}^{n} Q'_{ij}(z_k^2 - z_{k-1}^2)/2 = 0 \tag{10}$$

Bending stiffness terms are written in alternative form in [13] as

$$D_{ij\pm} = \zeta_{\pm}t^3/12 \times Q'_{ij\pm} \tag{11}$$

and

$$D_{ij\bigcirc} = \zeta_{\bigcirc}t^3/12 \times Q'_{ij\bigcirc} \tag{12}$$

respectively, because the number of plies ($n = 7$) is now related directly to the bending stiffness terms by the expression

$$\zeta_{\pm} + \zeta_{\bigcirc} = \zeta = n^3 \tag{13}$$

where $\zeta_{\pm} = 342$ and $\zeta = n^3 = 343$.

The stiffness parameters are hereby extended to include both cross plies $\zeta_{\bigcirc}$ and $\zeta_{\bullet}$, including percentage values to indicate the relative proportion ($n_{\pm}/n$, $n_{\bigcirc}/n$, and $n_{\bullet}/n$) and relative contribution to bending stiffness ($\zeta_{\pm}/\zeta$, $\zeta_{\bigcirc}/\zeta$, and $\zeta_{\bullet}/\zeta$) of each ply subsequence within the laminate (i.e., a subsequence containing either ±, ○, or ● plies). These relationships also help to identify other laminate properties, which are subsets of FOLs. For instance, a quasi-homogeneous orthotropic laminate (QHOL) has $n_{\pm}/n = \zeta_{\pm}/\zeta$, $n_{\bigcirc}/n = \zeta_{\bigcirc}/\zeta$, and $n_{\bullet}/n = \zeta_{\bullet}/\zeta$, respectively, whereas a $\pi/3$ FIL has $n_{\pm}/n = 2n_{\bigcirc}/n = \zeta_{\pm}/\zeta = 2\zeta_{\bigcirc}/\zeta$.

Parameters relating to ● cross plies have been omitted from the tables of stacking sequences that follow, but are readily calculated from

$$\begin{aligned} &n_{\bullet} = n - n_{\pm} - n_{\bigcirc} \qquad &&n_{\bullet}/n = \{n - n_{\pm} - n_{\bigcirc}\}/n \\ &\zeta_{\bullet} = \zeta - \zeta_{\pm} - \zeta_{\bigcirc} \qquad &&\zeta_{\bullet}/\zeta = \{\zeta - \zeta_{\pm} - \zeta_{\bigcirc}\}/\zeta \end{aligned} \tag{14}$$

Table 1 provides a summary of the number of fully orthotropic stacking sequences in the definitive list for up to 21 plies, together with cross references to Tables 2–12 of abridged sequences for upper- and lower-bound $\zeta_{\pm}$ values; these bounds have been demonstrated [15] to give upper- and lower-bound closed-form buckling solutions, respectively, in compression-loaded rectangular plates. Care must, however, be exercised when applying this rule to other planform geometries and load combinations.

The new stacking sequences have been validated against the published ESDU listing [13], and reference numbers with prefix designations *A* and *S* are included in Tables 4 and 12, respectively, for cross-referencing purposes. Note that sequences containing only angle plies may share the same value of $\zeta_{\pm}$, and therefore the order of such sequences may differ from the ESDU listing. For such laminates, the second prefix has no significance, but has been given *AS* or *SS* designations, which is justified in the sense that *AS* sequences 3, 9, 15, and 39 of Table 4 are of the general form

$$[+/-/-/+/*/-/+/+/-]$$

and *SS* sequences 53, 103, 213, 333, and 685 of Table 12 are of the general form

$$[+/-/-/+/-/+/+/-/*]_S$$

where $*$ represents any symmetric cross-ply sequence.

Furthermore, the ESDU data item [13] listed even- and odd-ply sequences separately, which explains the apparent inconsistency between the two numbering systems. Note also that Table 9 begins

Table 2 FOLs for 7–21-ply laminates corresponding to prefix designation *AC* for antisymmetric *A* angle plies and cross-symmetric *C* cross plies

Ref.	Sequence	n	$n_\pm$	$n_\bigcirc$	ζ	$\zeta_\pm$	$\zeta_\bigcirc$	$n_\pm/n$, %	$n_\bigcirc/n$, %	$\zeta_\pm/\zeta$, %	$\zeta_\bigcirc/\zeta$, %
*AC*1	+ ○ ● − − ● ○ ● ● ○ + + ○ ● −	15	6	4	3375	1782	796	40.0	26.7	52.8	23.6
⋮		⋮									
*AC*24	+ − − + ● ○ ○ ○ ● ● ○ − + + −	15	8	4	3375	3032	172	53.3	26.7	89.8	5.1
*AC*25	+ ○ ● − − ● ○ + − ● ○ + + ○ ● −	16	8	4	4096	2144	976	50.0	25.0	52.3	23.8
⋮		⋮									
*AC*36	+ − − + ● ○ ○ ● ○ ● ● ○ − + + −	16	8	4	4096	3584	256	50.0	25.0	87.5	6.3
*AC*37	+ ○ ● − ● ○ − − ● + + ● ○ + ○ ● −	17	8	4	4913	2264	1324	47.1	23.5	46.1	26.9
⋮		⋮									
*AC*120	+ + − − − ● ○ ○ ○ ● ● ○ + + + − −	17	10	4	4913	4570	172	58.8	23.5	93.0	3.5
*AC*121	+ ○ ● ○ ● − − ● − ○ ● + ○ + + ○ ● ○ ● −	20	8	6	8000	3008	2496	40.0	30.0	37.6	31.2
⋮		⋮									
*AC*127	+ ○ ● ● − ○ − ○ ● − + ○ ● + ● + ○ ○ ● −	20	8	6	8000	3200	2400	40.0	30.0	40.0	30.0
⋮		⋮									
*AC*210	+ − + − − ● + ○ ○ ○ ● ● ● − ○ + + − + −	20	12	4	8000	7296	352	60.0	20.0	91.2	4.4

with laminate designation $NN12$, which is explained by the fact that $_{+}NN_{\bullet}$ sequences are not shown, because they are identical to the $_{+}NN_{\bigcirc}$ listing with $\bullet$ and $\bigcirc$ cross plies interchanged. The first sequence (i.e., that with lowest $\zeta_\pm$ and $\zeta_\bigcirc$) is given by sequence $NN1$:

$$[+/-/\bullet/\bullet/\bullet/\bullet/-/+/-/\bullet/+/+/\bullet/-/\bullet]_T$$

which is of the form $_{+}NN_{\bullet}$.

For each unique stacking sequence with the prefix *SN*, denoting symmetric angle plies and nonsymmetric cross plies, alternative sequences also exist, because each cross ply in the subsequence is interchangeable, $\bigcirc$ with $\bullet$, and the order of these alternative nonsymmetric subsequences are themselves reversible. For each laminate with either the prefix *SC* or *SS*, denoting symmetric angle plies with cross-symmetric or symmetric cross plies, respectively, an alternative arises again from the interchangeability of each cross ply ($\bigcirc$ with $\bullet$) in the subsequence.

By contrast, the sequences presented in [6,7,10,16] have been algorithmically filtered to provide mathematically and mechanically unique sequences. Applying this filtering to the 18 FILs, with $\pi/3$ isotropy, identified as a subset of FOLs, which are of the form (and number) $_{+}NN_{+}$ (2), $_{+}NN_{-}$ (8) and $_{+}NN_{\bigcirc}$ (8), and correspond to sequences numbers $NN1071$—$NN1088$ in the definitive list [3], reveals that only 1 of the 2 $_{+}NN_{+}$ sequences is unique when the order is reversed, only 4 from the 8 $_{+}NN_{-}$ sequences are unique when the order is reversed and $+$ and $-$ plies are interchanged, and $_{+}NN_{\bigcirc}$ sequences are identical to $_{+}NN_{-}$ sequences with $-$ and $\bigcirc$ plies interchanged, leaving the 5 mathematically and mechanically unique sequences identified in [7], corresponding to $NN1071$, 1073–1075, and 1077, respectively,

$$[\pm/-/\bigcirc_3/+_2/\bigcirc/\mp/\pm/-_2/\bigcirc_2/+]_T$$

$$[+/\bigcirc/-_2/\bigcirc/+/\bigcirc/\mp/\pm/\bigcirc/\mp/\bigcirc_2/\pm]_T$$

$$[+/\bigcirc/-/\bigcirc/\mp_2/\bigcirc/-/\bigcirc/+/\pm/\bigcirc_2/\pm]_T$$

$$[+/\bigcirc_2/-_2/\pm/\mp/+/\bigcirc_3/\pm/\bigcirc/\pm]_T$$

$$[+/\bigcirc/-/\bigcirc/\mp_2/\bigcirc/\mp/\bigcirc_2/\mp/+/\bigcirc/-]_T$$

where $+/-/\bigcirc$ represent $60/-60/0$ deg, respectively, or indeed any angle combination with $\pi/3$ separation.

Table 3 FOLs for 7–21-ply laminates corresponding to prefix designation *AN* for antisymmetric *A* angle plies and nonsymmetric *N* cross plies

Ref.	Sequence	n	$n_\pm$	$n_\bigcirc$	ζ	$\zeta_\pm$	$\zeta_\bigcirc$	$n_\pm/n$, %	$n_\bigcirc/n$, %	$\zeta_\pm/\zeta$, %	$\zeta_\bigcirc/\zeta$, %
*AN*1	+ ○ ● − − ● ● ● ○ ● + + ○ ● −	15	6	3	3375	1782	747	40.0	20.0	52.8	22.1
⋮		⋮									
*AN*24	+ − ● ○ ○ ○ − ○ + ● ● ○ ○ + −	15	6	6	3375	2070	846	40.0	40.0	61.3	25.1
*AN*25	+ ○ ● ● − − ● − ● + ○ + + ○ ● ● −	17	8	3	4913	2168	939	47.1	17.6	44.1	19.1
⋮		⋮									
*AN*256	+ − − + ○ ● ○ ○ ○ ● ● ○ ○ − + + −	17	8	6	4913	4184	558	47.1	35.3	85.2	11.4
*AN*257	+ ● ○ ● − − ● ○ − + ● ○ + + ○ ● ● −	18	8	4	5832	2528	976	44.4	22.2	43.3	16.7
⋮		⋮									
*AN*388	+ − − + ○ ● ○ ○ ● ○ ● ● ○ ○ − + + −	18	8	6	5832	4832	744	44.4	33.3	82.9	12.8
*AN*389	+ ● ● ○ ● − − − ● ● ○ + + + ○ ● ● ● −	19	8	3	6859	2648	747	42.1	15.8	38.6	10.9
⋮		⋮									
*AN*2372	+ + − − ○ − ● ○ ○ ○ ● ● ○ + ○ + + − −	19	10	6	6589	5914	774	52.6	31.6	86.2	11.3
*AN*2373	+ ● ○ ● ● − − ● − ○ ● + ○ + + ○ ● ● ● −	20	8	4	8000	3008	1120	40.0	20.0	37.6	14.0
⋮		⋮									
*AN*3292	+ − − + ○ ○ ● ○ ○ ● ○ ● ● ○ ○ ○ − + + −	20	8	8	8000	6272	1472	40.0	40.0	78.4	18.4
*AN*3293	+ ● ○ ● ● ○ − − − − ● + + + + ● ○ ○ ● ● −	21	10	4	9261	3130	2092	47.6	19.0	33.8	22.6
⋮		⋮									
*AN*14532	+ − + − − ○ + ● ○ ○ ○ ● ● ○ − ○ + + − + −	21	12	6	9261	8316	774	57.1	28.6	89.8	8.4

Table 4 FOLs for 7–21-ply laminates corresponding to prefix designation *AS* for antisymmetric *A* angle plies and symmetric *S* cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %	ESDU 82013 [13]
*AS*1	+ − − ● + + −	7	6	0	343	342	0	85.7	0.0	99.7	0.0	——
*AS*2	+ − − ○ + + −	7	6	1	343	342	1	85.7	14.3	99.7	0.3	*A*381
*AS*3	+ − − + − + + −	8	8	0	512	512	0	100.0	0.0	100.0	0.0	*A*1
*AS*4	+ − ● − ● + ● + −	9	6	0	729	630	0	66.7	0.0	86.4	0.0	——
⋮		⋮										
*AS*9	+ − − + ○ − + + −	9	8	1	729	728	1	66.7	33.3	86.4	13.6	*A*383
*AS*10	+ ● − − − + + + ● −	10	8	0	1000	704	0	80.0	0.0	70.4	0.0	——
⋮		⋮										
*AS*15	+ − − + ○ ○ − + + −	10	8	2	1000	992	8	80.0	20.0	70.4	29.6	*A*4
*AS*16	+ ● − − ● ● ● + + ● −	11	6	0	1331	918	0	54.5	0.0	69.0	0.0	——
⋮		⋮										
*AS*39	+ − − + ○ ○ ○ − + + −	11	8	3	1331	1304	27	72.7	27.3	98.0	2.0	*A*387
*AS*40	+ ● − ● − − + + ● + ● −	12	8	0	1728	1088	0	66.7	0.0	63.0	0.0	——
⋮		⋮										
*AS*60	+ − + − − − + + + − + −	12	12	0	1728	1728	0	100.0	0.0	100.0	0.0	*A*10
*AS*61	+ ● ● − − − ● + + + ● ● −	13	8	0	2197	1208	0	61.5	0.0	55.0	0.0	——
⋮		⋮										
*AS*144	+ − + − − ○ ○ ○ + + − + −	13	10	3	2197	2170	27	76.9	23.1	98.8	1.2	*A*397
*AS*145	+ ● ● − − ● − + ● + + ● ● −	14	8	0	2744	1472	0	57.1	0.0	53.6	0.0	——
⋮		⋮										
*AS*220	+ + − − − − ○ ○ + + + + − −	14	12	2	2744	2736	8	85.7	14.3	99.7	0.3	*A*24
*AS*221	+ ● ● − ● − − ● + + ● + ● ● −	15	8	0	3375	1688	0	53.3	0.0	50.0	0.0	——
⋮		⋮										
*AS*508	+ + − − − − + ○ − + + + + − −	15	14	1	3375	3374	1	93.3	6.7	100.0	0.0	*A*422
*AS*509	+ ● ● ● − − − ● ● + + + ● ● ● −	16	8	0	4096	1856	0	50.0	0.0	45.3	0.0	——
⋮		⋮										
*AS*776	+ + − − − − + + − − + + + + − −	16	16	0	4096	4096	0	100.0	0.0	100.0	0.0	*A*56
*AS*777	+ ● ● ● − − ● − ● + ● + + ● ● ● −	17	8	0	4913	2168	0	47.1	0.0	44.1	0.0	——
⋮		⋮										
*AS*1778	+ + − − − + − − ○ + + − + + + − −	17	16	1	4913	4912	1	94.1	5.9	100.0	0.0	*A*484
*AS*1779	+ ● ● ● − ● − − ● ● + + ● + ● ● ● −	18	8	0	5832	2432	0	44.4	0.0	41.7	0.0	——
⋮		⋮										
*AS*2712	+ + − − − − + + ○ ○ − − + + + + − −	18	16	2	5832	5824	8	88.9	11.1	99.9	0.1	*A*144
*AS*2713	+ ● ● ● ● − − − ● ● ● + + + ● ● ● ● −	19	8	0	6859	2648	0	42.1	0.0	38.6	0.0	——
⋮		⋮										
*AS*6224	+ + − − + − − − ○ ○ ○ + + + − + + − −	19	16	3	6859	6832	27	84.2	15.8	99.6	0.4	*A*652
*AS*6225	+ ● ● ● ● − − ● − ● ● + ● + + ● ● ● ● −	20	8	0	8000	3008	0	40.0	0.0	37.6	0.0	——
⋮		⋮										
*AS*9514	+ + − + − − − − − + − + + + + + − + − −	20	20	0	8000	8000	0	100.0	0.0	100.0	0.0	*A*373
*AS*9515	+ ● ● ● ● ● − − − − ● + + + + ● ● ● ● ● −	21	10	0	9261	3130	0	47.6	0.0	33.8	0.0	——
⋮		⋮										
*AS*21906	+ + + − − − − − − ○ ○ ○ + + + + + + − − −	21	18	3	9261	9234	27	85.7	14.3	99.7	0.3	——

Sequence *AC*127, given in Table 2, also has special significance: it possesses quasi-homogeneous properties and verifies the only sequence containing all four (+/ − /○/●) ply orientations, presented in [16] for a 20-ply laminate satisfying Eqs. (3) and (5), but not Eq. (4). Note that *AC*127 and *AC*128 are identical when the cross plies are interchanged, ○ with ●.

The mixing of laminate sequences has previously been considered as a means to develop laminates of different form. For instance, Caprino and Crivelli-Visconti [8] demonstrated that a fully orthotropic angle-ply laminate could be obtained by stacking two balanced and symmetric semilaminates (i.e., $[\bigcirc/+/-]_S$ and $[\bigcirc/-/+]_S$), to give $[\bigcirc/+/-/-/+/\bigcirc]_A$, the simplest form of which is $(\pm)_S/(\mp)_S$ and corresponds to the 8-ply laminate *AS*3 given in Table 4. It should be noted that each sequence defined in Tables 2–12 with up to 21 plies also represents the antisymmetric or symmetric half of an even-ply laminate with up to 42 plies. This has ironic significance in the development of the FILs described previously: Reference [17] presented stacking sequences representing the symmetric halves of 36-ply FILs, regarded as being the minimum number of plies for $\pi/3$ FILs until the publication of the 18-ply sequences in [7]. However, because of the assumptions of symmetry, the authors [17] failed to realize that 9 of the 89 sequences listed are, in fact, 18-ply FILs.[†]

By contrast, Bartholomew [12] identified 7 FOLs with nonsymmetric form by combining symmetric and antisymmetric sequences using a method referred to as *interlocking*, in which the cross plies of one compatible sequence are substituted by the corresponding angle plies from another; for example, combining *AS*43 and *SS*36 from Tables 4 and 12, respectively, produces laminate *NS*2 from Table 5:

*AS*43			+	○	−	○	−	−	+	+	○	+	○	−		
*SS*36	+	−	○	−	○	+	○	○	○	○	+	○	−	○	−	+
*NS*2	+	−	+	−	−	+	−	−	+	+	+	+	−	−	−	+

[†]The 18-ply FILs are listed as sequence numbers 57, 65, 66, 69, 70, 73, and 76–78.

Table 5 FOLs for 7–21-ply laminates corresponding to prefix designation $_{+}NS_{+}$ for nonsymmetric N angle plies and symmetric S cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
$NS1$	+ − − − + + + + − − + − − + − +	16	16	0	4096	4096	0	100.0	0.0	100.0	0.0
$NS2$	+ − + − − + − − + + + + − − − +	16	16	0	4096	4096	0	100.0	0.0	100.0	0.0
$NS3$	+ − ● − − + + + ● + − − − + ● − +	17	14	0	4913	4046	0	82.4	0.0	82.4	0.0
⋮		⋮									
$NS14$	+ − + − − − + + ○ + − + − + − − +	17	16	1	4913	4912	1	94.1	5.9	100.0	0.0
$NS15$	+ ● − − − + + − + + + − − + − − ● +	18	16	0	5832	4480	0	88.9	0.0	76.8	0.0
⋮		⋮									
$NS32$	+ + − − ○ − − − + + + + + ○ − − − +	18	16	2	5832	5344	488	88.9	11.1	91.6	8.4
$NS39$	+ ● − − ● − + + + ● − + − + ● − − ● +	19	14	0	6859	4718	0	73.7	0.0	68.8	0.0
⋮		⋮									
$NS124$	+ + − − − − ○ + + ○ − + ○ + + − − − +	19	16	3	6859	6640	219	84.2	15.8	96.8	3.2
$NS143$	+ − ● ● − − + + + − + + − − − + ● ● − +	20	16	0	8000	5632	0	80.0	0.0	70.4	0.0
⋮		⋮									
$NS300$	+ − + − + − − − + − + + + + − − + − − +	20	20	0	8000	8000	0	100.0	0.0	100.0	0.0
$NS365$	+ − − ● + − − + + + ● + + − − − − ● − + +	21	18	0	9261	8082	0	85.7	0.0	87.3	0.0
⋮		⋮									
$NS514$	+ + − − − + − − + ○ ○ ○ + + − + + − − − +	21	18	3	9261	9234	27	85.7	14.3	99.7	0.3

Table 6 FOLs for 7–21-ply laminates corresponding to prefix designation $_{+}NS_{-}$ for nonsymmetric N angle plies and symmetric S cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
$NS17$	+ ● − + − − − + + − + + − − + + ● −	18	16	0	5832	4480	0	88.9	0.0	76.8	0.0
⋮		⋮									
$NS38$	+ − + − + − − ○ + − ○ − + + + + − −	18	16	2	5832	5776	56	88.9	11.1	99.0	1.0
$NS71$	+ + − − ● − − ● + ● + ● + − ● + − + −	19	14	0	6859	6158	0	73.7	0.0	89.8	0.0
⋮		⋮									
$NS126$	+ − + − − + ○ + − ○ − − ○ + + + + − −	19	16	3	6859	6640	219	84.2	15.8	96.8	3.2
$NS127$	+ ● ● − + − − − + − + + + − − + + ● ● −	20	16	0	8000	4912	0	80.0	0.0	61.4	0.0
⋮		⋮									
$NS308$	+ − − + + − + − + − − − + − + + + + − −	20	20	0	8000	8000	0	100.0	0.0	100.0	0.0
$NS309$	+ ● + − ● − − − − + ● + + + − + ● − + ● −	21	16	0	9261	6448	0	76.2	0.0	69.6	0.0
⋮		⋮									
$NS516$	+ − − + + + − − − ○ ○ ○ − + + − + + + − −	21	18	3	9261	9234	27	85.7	14.3	99.7	0.3

This technique is also applied in the ESDU data item [13], in which listings of interlocking sequences are presented. The listings reveal that there are no asymmetric sequences with less than 16 plies; the single asymmetric sequence corresponds to $NS2$, derived previously. Additionally, the tables reveal that there are 6 and 28 sequences with 18- and 20-ply asymmetric sequences, respectively; errors in the referencing of interlocking sequences with odd numbers of layers make the exact number of asymmetric sequences (amounting to 2, 7, and 5 sequences with 17, 19, and 21 plies, respectively) difficult to verify independently.‡

III. Calculation of Membrane and Bending Stiffness Terms

The calculation procedure for the elements A_{ij} and D_{ij} of the extensional **A** and bending **D** stiffness matrices using the dimensionless parameters provided in Tables 2–9 are as follows:

$$A_{ij} = \{n_{\pm}/2 \times Q'_{ij+} + n_{\pm}/2 \times Q'_{ij-} + n_{\bigcirc} Q'_{ij\bigcirc} + n_{\bullet} Q'_{ij\bullet}\} \times t \tag{15}$$

$$D_{ij} = \{\zeta_{\pm}/2 \times Q'_{ij+} + \zeta_{\pm}/2 \times Q'_{ij-} + \zeta_{\bigcirc} Q'_{ij\bigcirc} + \zeta_{\bullet} Q'_{ij\bullet}\} \times t^3/12 \tag{16}$$

‡The error in [13] relates specifically to the first 14 lines of Table 9.7, where $S382$, $S386$, $S392$, $S395$, $S396$, $S398$, and $S403$ should read $S50$, $S54$, $S60$, $S63$, $S64$, $S66$, and $S71$, respectively.

The form of Eqs. (15) and (16) was chosen because they are readily modified to account for laminates with extensional and bending anisotropy by replacing $n_{\pm}/2 \times Q'_{ij+}$ with $n_{\pm}(n_{+}/n_{\pm})Q'_{ij+}$ and $n_{\pm}/2 \times Q'_{ij-}$ with $n_{\pm}(1 - n_{+}/n_{\pm})Q'_{ij-}$ and by replacing $\zeta_{\pm}/2 \times Q'_{ij+}$ with $\zeta_{\pm}(\zeta_{+}/\zeta_{\pm}) \times Q'_{ij+}$ and $\zeta_{\pm}/2 \times Q'_{ij-}$ with $\zeta_{\pm}(1 - \zeta_{+}/\zeta_{\pm}) \times Q'_{ij-}$. The use of these modified equations requires the calculation of additional stiffness parameters n_{+} and ζ_{+}, relating to the extensional and bending stiffness contribution of positive (θ) angle plies, respectively.

The transformed reduced stiffness terms in Eqs. (15) and (16) are given by

$$\begin{aligned}
Q'_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\sin^4\theta \\
Q'_{12} &= Q'_{21} = (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta\sin^2\theta \\
&\quad + Q_{12}(\cos^4\theta + \sin^4\theta) \\
Q'_{16} &= Q'_{61} = \{(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} \\
&\quad + 2Q_{66})\sin^2\theta\} \cos\theta\sin\theta \\
Q'_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\cos^4\theta \\
Q'_{26} &= Q'_{62} = \{(Q_{11} - Q_{12} - 2Q_{66})\sin^2\theta + (Q_{12} - Q_{22} \\
&\quad + 2Q_{66})\cos^2\theta\} \cos\theta\sin\theta \\
Q'_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta \\
&\quad + Q_{66}(\cos^4\theta + \sin^4\theta)
\end{aligned} \tag{17}$$

and the reduced stiffness terms are given by

Table 7 FOLs for 7–21-ply laminates corresponding to prefix designation $_{+}NN_{+}$ for nonsymmetric N angle plies and nonsymmetric N cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
$NN33$	+ ● − − − + + + − ● + ● − − +	15	12	0	3375	2700	0	80.0	0.0	80.0	0.0
⋮		⋮									
$NN36$	+ ○ − − − + + + − ○ + ○ − − +	15	12	3	3375	2700	675	80.0	20.0	80.0	20.0
$NN65$	+ ● − − ● − + + + + − − ● ● − +	16	12	0	4096	2832	0	75.0	0.0	69.1	0.0
⋮		⋮									
$NN70$	+ ○ − − ○ − + + + + − − ○ ○ − +	16	12	4	4096	2832	1264	75.0	25.0	69.1	30.9
$NN457$	+ ● − ● − ● − + + + ● ● ● − ● − +	17	10	0	4913	3130	0	58.8	0.0	63.7	0.0
⋮		⋮									
$NN722$	+ − + − − ○ ○ ○ + ○ + − + ○ − − +	17	12	5	4913	4428	485	70.6	29.4	90.1	9.9
$NN935$	+ ● − − ● ● ● + ● ● + ● ● − ● ● − +	18	8	0	5832	3584	0	44.4	0.0	61.5	0.0
⋮		⋮									
$NN1258$	+ − − + ○ ○ − + ○ ○ ○ + − + − ○ − +	18	12	6	5832	4896	936	66.7	33.3	84.0	16.0
$NN3291$	+ − ● ● − ● ● + ● + − + ● − ● ● ● − +	19	10	0	6859	4090	0	52.6	0.0	59.6	0.0
⋮		⋮									
$NN4944$	+ − + − − + ○ − + − ○ ○ + + + − − − +	19	16	3	6859	6688	171	84.2	15.8	97.5	2.5
$NN6041$	+ ● − ● ● − − + ● + + − + ● ● − ● ● − +	20	12	0	8000	4656	0	60.0	0.0	58.2	0.0
⋮		⋮									
$NN8778$	+ + − − − − + ○ ○ + ○ − + ○ + + − − − +	20	16	4	8000	7744	256	80.0	20.0	96.8	3.2
$NN21309$	+ ● − − ● ● ● ● ● + + + − ● ● ● − − ● ● +	21	10	0	9261	4858	0	47.6	0.0	52.5	0.0
⋮		⋮									
$NN40762$	+ + − − − − + ○ + − + ○ ○ − + + + − − − +	21	18	3	9261	9090	171	85.7	14.3	98.2	1.8

$$
\begin{aligned}
&Q_{11} = E_1/(1-\nu_{12}\nu_{21}) \\
&Q_{12} = \nu_{12}E_2/(1-\nu_{12}\nu_{21}) = \nu_{21}E_1/(1-\nu_{12}\nu_{21}) \\
&Q_{22} = E_2/(1-\nu_{12}\nu_{21}) \qquad Q_{66} = G_{12}
\end{aligned} \tag{18}
$$

For optimum design of angle-ply laminates, lamination parameters are often preferred, because these allow the stiffness terms to be expressed as linear variables. The optimized lamination parameters may then be matched against a corresponding set of laminate stacking sequences. In the context of the parameters presented in the current paper, the four lamination parameters are related through the following expressions:

$$
\begin{aligned}
\xi_1^A = \xi_1 = \{&n_{\pm}(n_+/n_{\pm})\cos(2\theta_+) + n_{\pm}(1-n_+/n_{\pm})\cos(2\theta_-) \\
&+ n_{\bigcirc}\cos(2\theta_{\bigcirc}) + n_{\bullet}\cos(2\theta_{\bullet})\}/n \\
\xi_2^A = \xi_2 = \{&n_{\pm}(n_+/n_{\pm})\cos(4\theta_+) + n_{\pm}(1-n_+/n_{\pm})\cos(4\theta_-) \\
&+ n_{\bigcirc}\cos(4\theta_{\bigcirc}) + n_{\bullet}\cos(4\theta_{\bullet})\}/n
\end{aligned} \tag{19}
$$

and

$$
\begin{aligned}
\xi_1^D = \xi_9 = \{&\zeta_{\pm}(\zeta_+/\zeta_{\pm})\cos(2\theta_+) + \zeta_{\pm}(1-\zeta_+/\zeta_{\pm})\cos(2\theta_-) \\
&+ \zeta_{\bigcirc}\cos(2\theta_{\bigcirc}) + \zeta_{\bullet}\cos(2\theta_{\bullet})\}/\zeta \\
\xi_2^D = \xi_{10} = \{&\zeta_{\pm}(\zeta_+/\zeta_{\pm})\cos(4\theta_+) + \zeta_{\pm}(1-\zeta_+/\zeta_{\pm})\cos(4\theta_-) \\
&+ \zeta_{\bigcirc}\cos(4\theta_{\bigcirc}) + \zeta_{\bullet}\cos(4\theta_{\bullet})\}/\zeta
\end{aligned} \tag{20}
$$

where the bending stiffness parameter $\zeta_+ = \zeta_{\pm}/2$ for FOLs, EILs, and FILs; hence, Eqs. (20) reduce to

$$
\begin{aligned}
\xi_1^D &= \xi_9 = \{\zeta_{\pm}\cos(2\theta_{\pm}) + \zeta_{\bigcirc}\cos(2\theta_{\bigcirc}) + \zeta_{\bullet}\cos(2\theta_{\bullet})\}/\zeta \\
\xi_2^D &= \xi_{10} = \{\zeta_{\pm}\cos(4\theta_{\pm}) + \zeta_{\bigcirc}\cos(4\theta_{\bigcirc}) + \zeta_{\bullet}\cos(4\theta_{\bullet})\}/\zeta
\end{aligned} \tag{21}
$$

Elements of the extensional and bending stiffness matrices are related to the lamination parameters, respectively, by

$$
\begin{aligned}
A_{11} &= \{U_1 + \xi_1 U_2 + \xi_2 U_3\} \times H \\
A_{12} &= A_{21} = \{-\xi_2 U_3 + U_4\} \times H \\
A_{22} &= \{U_1 - \xi_1 U_2 + \xi_2 U_3\} \times H \\
A_{66} &= \{-\xi_2 U_3 + U_5\} \times H
\end{aligned} \tag{22}
$$

and

$$
\begin{aligned}
D_{11} &= \{U_1 + \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12 \\
D_{12} &= D_{21} = \{U_4 - \xi_{10} U_3\} \times H^3/12 \\
D_{22} &= \{U_1 - \xi_9 U_2 + \xi_{10} U_3\} \times H^3/12 \\
D_{66} &= \{-\xi_{10} U_3 + U_5\} \times H^3/12
\end{aligned} \tag{23}
$$

where the laminate invariants are given in terms of the reduced stiffnesses of Eq. (18) by

$$
\begin{aligned}
U_1 &= \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8 \\
U_2 &= \{Q_{11} - Q_{22}\}/2 \\
U_3 &= \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8 \\
U_4 &= \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8 \\
U_5 &= \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8
\end{aligned} \tag{24}
$$

In [14], a modified set of lamination parameters was adopted for the purposes of optimum design, because bounds are then defined by the parabola $\xi_{10} = \xi_9^2$ with limits $-1 \le \xi_9 \le 1$ and $0 \le \xi_9^2$; hence, Eqs. (21) may be rewritten as

$$
\begin{aligned}
\xi_1^D &= \xi_9 = \{\zeta_{\pm}\cos(2\theta_{\pm}) + \zeta_{\bigcirc}\cos(2\theta_{\bigcirc}) + \zeta_{\bullet}\cos(2\theta_{\bullet})\}/\zeta \\
\xi_2^D &= \xi_{10} = \{\zeta_{\pm}\cos^2(2\theta_{\pm}) + \zeta_{\bigcirc}\cos^2(2\theta_{\bigcirc}) + \zeta_{\bullet}\cos^2(2\theta_{\bullet})\}/\zeta
\end{aligned} \tag{25}
$$

Figures 1a–1k illustrate the feasible domains of lamination parameters in the form of Eqs. (25) for all FOLs from the definitive list, in which the $+/-/\bigcirc/\bullet$ of Tables 2–12 correspond to $45/-45/0/90$ deg, respectively. Additionally Fig. 1l provides the feasible domain for the corresponding subset of EILs with $\pi/3$ isotropy in which $+/-/\bigcirc$ now represent $60/-60/0$ deg,

Table 8 FOLs for 7–21-ply laminates corresponding to prefix designation $_{+}NN_{-}$ or nonsymmetric N angle plies and nonsymmetric N cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
*NN*67	+ − ● ● − + + − − + − ● + + ● −	16	12	0	4096	2832	0	75.0	0.0	69.1	0.0
⋮		⋮									
*NN*94	+ − − ○ + + ○ − − + ○ ○ − + + −	16	12	4	4096	3600	496	75.0	25.0	87.9	12.1
*NN*299	+ ● ● − − ● + ● − − + + ● + ● ● −	17	10	0	4913	2554	0	58.8	0.0	52.0	0.0
⋮		⋮									
*NN*714	+ − ○ + − − + − + − ○ + ○ + − + −	17	14	3	4913	4238	675	82.4	17.6	86.3	13.7
*NN*991	+ − ● ● + ● − − − + ● + − + + ● ● −	18	12	0	5832	3600	0	66.7	0.0	61.7	0.0
⋮		⋮									
*NN*1268	+ − − + ○ + − ○ ○ ○ − ○ ○ + + − + −	18	12	6	5832	5328	504	66.7	33.3	91.4	8.6
*NN*1403	+ ● ● − ● ● ● − − + ● ● ● + + ● ● ● −	19	8	0	6859	2936	0	42.1	0.0	42.8	0.0
⋮		⋮									
*NN*4940	+ − − + + ○ − + − − ○ − ○ + + + + − −	19	16	3	6859	6544	315	84.2	15.8	95.4	4.6
*NN*5377	+ ● − ● ● ● − + + − − − ● + + + ● ● ● −	20	12	0	8000	3888	0	60.0	0.0	48.6	0.0
⋮		⋮									
*NN*8780	+ − + − − + ○ + − ○ − ○ ○ − + + + + − −	20	16	4	8000	7744	256	80.0	20.0	96.8	3.2
*NN*10839	+ ● ● ● − ● − ● + − ● − ● + + + ● ● ● ● −	21	10	0	9261	3706	0	47.6	0.0	40.0	0.0
⋮		⋮									
*NN*40768	+ − + − + − − + ○ ○ − − + ○ + − + + + − −	21	18	3	9261	9090	171	85.7	14.3	98.2	1.8

Table 9 FOLs for 7–21-ply laminates corresponding to prefix designation $_{+}NN_{\bigcirc}$ for nonsymmetric N angle plies and nonsymmetric N cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
*NN*12	+ ● − ○ − ○ ○ + ● − + + ● − ○	15	8	4	3375	1784	844	53.3	26.7	52.9	25.0
⋮		⋮									
*NN*32	+ − ○ ○ − ○ ○ + + ○ ○ − − + ○	15	8	7	3375	2072	1303	53.3	46.7	61.4	38.6
*NN*43	+ − ● ○ ○ ● − + ○ − ● + + ● − ○	16	8	4	4096	2144	1072	50.0	25.0	52.3	26.2
⋮		⋮									
*NN*74	+ − ○ + − − ○ ○ + − + + − + − ○	16	12	4	4096	3024	1072	75.0	25.0	73.8	26.2
*NN*105	+ − ● ○ ● ○ ● − ● + − + + ● − ● ○	17	8	3	4913	2168	1179	47.1	17.6	44.1	24.0
⋮		⋮									
*NN*650	+ − + ○ − ○ − ○ − ○ + + + + − − ○	17	12	5	4913	3708	1205	70.6	29.4	75.5	24.5
*NN*737	+ − ● ● ○ ○ ● − ○ ● + + − + − ● ● ○	18	8	4	5832	2432	1264	44.4	22.2	41.7	21.7
⋮		⋮									
*NN*1236	+ − + − ○ − ○ ○ ○ ○ + + + − − − + ○	18	12	6	5832	4608	1224	66.7	33.3	79.0	21.0
*NN*1289	+ ● ● − ○ ○ − ● ● ● − + + + ● ● ● − ○	19	8	3	6859	2648	1467	42.1	15.8	38.6	21.4
⋮		⋮									
*NN*4544	+ − + − − ○ ○ ○ + ○ + − + − + − − + ○	19	14	5	6859	5534	1325	73.7	26.3	80.7	19.3
*NN*4958	+ ● − ● ● ○ ○ − ○ ● ● + − + + ● ● − ● ○	20	8	4	8000	3008	1504	40.0	20.0	37.6	18.8
⋮		⋮									
*NN*8380	+ + − − − ○ ○ + ○ − − + + − + + + − − ○	20	16	4	8000	6496	1504	80.0	20.0	81.2	18.8
*NN*8807	+ ● − ● ○ ● ○ ● − − ● + + + + − − ● ● ● ○	21	10	3	9261	3130	1827	47.6	14.3	33.8	19.7
⋮		⋮									
*NN*39424	+ − − + + − ○ ○ ○ ○ − + − + − + + − + − ○	21	16	5	9261	7696	1565	76.2	23.8	83.1	16.9

Table 10 FOLs for 7–21-ply laminates corresponding to prefix designation *SC* for symmetric *S* angle plies and cross-symmetric *C* cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
*SC*1	+ − ○ − ● + ● ● ○ ○ + ○ − ● − +	16	8	4	4096	3008	544	50.0	25.0	73.4	13.3
*SC*2	+ − ● − ○ + ○ ○ ● ● + ● − ○ − +	16	8	4	4096	3008	544	50.0	25.0	73.4	13.3
*SC*3	+ − ○ ● − ● ○ + ● + ● ○ − ○ ● − +	17	8	4	4913	3128	892	47.1	23.5	63.7	18.2
⋮		⋮									
*SC*6	+ − ● ○ − ○ ● + ○ + ○ ● − ● ○ − +	17	8	5	4913	3128	893	47.1	29.4	63.7	18.2
*SC*7	+ − ○ ● ○ − ● ● + ● ○ + ○ ○ − ● ○ ● − +	20	8	6	8000	4448	1776	40.0	30.0	55.6	22.2
⋮		⋮									
*SC*12	+ − ● ○ − ○ + ● ● ○ ● ○ ○ + ● − ● ○ − +	20	8	6	8000	4928	1536	40.0	30.0	61.6	19.2

Table 11 FOLs for 7–21-ply laminates corresponding to prefix designation *SN* for symmetric *S* angle plies and nonsymmetric *N* cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %
*SN*1	+ − ●○ − ●● + ● + ○○ − ●● − +	17	8	3	4913	3128	459	47.1	17.6	63.7	9.3
⋮		⋮									
*SN*8	+ − ○● − ○○ + ○ + ●● − ○○ − +	17	8	6	4913	3128	1326	47.1	35.3	63.7	27.0
*SN*9	+ ● − − ○●●○ + + ●○○● − − ● +	18	8	4	5832	3488	496	44.4	22.2	59.8	8.5
⋮		⋮									
*SN*16	+ ○ − − ●○○● + + ○●●○ − − ○ +	18	8	6	5832	3488	1848	44.4	33.3	59.8	31.7
*SN*17	+ ● − − ●○● + ● ● ○ + ○●● − − ● +	19	8	3	6859	4088	315	42.1	15.8	59.6	4.6
⋮		⋮									
*SN*80	+ ○ − − ○●○ + ○ ○ ● + ●○○ − − ○ +	19	8	8	6859	4088	2456	42.1	42.1	59.6	35.8
*SN*81	+ − ●●○ − ●● + ○ ● + ○○ − ●●● − +	20	8	4	8000	4448	592	40.0	20.0	55.6	7.4
⋮		⋮									
*SN*168	+ − ○○ − ● + ○○○ ●●● + ○ − ○○ − +	20	8	8	8000	4928	2720	40.0	40.0	61.6	34.0
*SN*169	+ ● − ●○ − − ○ + + ● + + ● − − ●● − ○ +	21	12	3	9261	5052	1515	57.1	14.3	54.6	16.4
⋮		⋮									
*SN*192	+ − − ○ + + − ●○○ ○ ●●○ − + + ○ − − +	21	12	6	9261	7740	1350	57.1	28.6	83.6	14.6

respectively. Stacking sequences lying along the broken line drawn between $(\xi_1^D, \xi_2^D) = (0, 0)$ and (1,1) in Fig. 1a consist of 0 and ±45 deg plies only, whereas those along the line between (0, 0) and (−1, 1) consist of ±45 and 90 deg plies only. Similarly, any stacking sequences corresponding to the points (−1, 1), (0, 0), or (1, 1), would, respectively, contain 90 deg plies, ±45 deg plies, or 0 deg plies only. In Fig. 1l, any sequence corresponding to point (−0.5, 0.25) would contain ±60 deg plies only. Finally, the corresponding subset of FILs with $\pi/3$ isotropy, identified in the previous section, correspond to the point (0, 0.5).

All feasible domains of FOLs have symmetry about the vertical (ξ_2^D) axis, with the exception of Fig. 1k, in which there is a bias toward the right-hand region of the graph. This is explained by the fact that ${}_{+}NN_{\bigcirc}$ laminates possess an angle ply on one surface and a cross ply on the other; hence, cross plies have a significant effect on the overall bending stiffness contribution, causing the bias toward the top right-hand corner of the feasible domain, representing a laminate with 0 deg plies only. The feasible domain for ${}_{+}NN_{\bullet}$ laminates is the mirror image, with a bias toward the left-hand region of the graph.

IV. Example Calculations

For IM7/8552 carbon-fiber/epoxy material with Young's moduli $E_1 = 161.0$ GPa and $E_2 = 11.38$ GPa, shear modulus $G_{12} = 5.17$ GPa and Poison ratio $\nu_{12} = 0.38$, lamina thickness $t =$

Table 12 FOLs for 7–21-ply laminates corresponding to prefix designation *SS* for symmetric *S* angle plies and symmetric *S* cross plies

Ref.	Sequence	n	$n_{\pm}$	$n_{\bigcirc}$	ζ	$\zeta_{\pm}$	$\zeta_{\bigcirc}$	$n_{\pm}/n$, %	$n_{\bigcirc}/n$, %	$\zeta_{\pm}/\zeta$, %	$\zeta_{\bigcirc}/\zeta$, %	ESDU 82013 [13]
*SS*1	+ − − ● + ● ● + ● − − +	12	8	0	1728	1568	0	66.7	0.0	90.7	0.0	——
⋮		⋮										
*SS*4	+ − − ○ + ○ ○ + ○ − − +	12	8	4	1728	1568	160	66.7	33.3	90.7	9.3	*S*1
*SS*5	+ − ● − ●● + + ●● − ● − +	14	8	0	2744	2048	0	57.1	0.0	74.6	0.0	——
⋮		⋮										
*SS*16	+ − − + − + ○ ○ + − + − − +	14	12	2	2744	2736	8	85.7	14.3	99.7	0.3	*S*4
*SS*17	+ − − ● + + − ● − + + ● − − +	15	12	0	3375	2988	0	80.0	0.0	88.5	0.0	——
*SS*20	+ − − ○ + + − ○ − + + ○ − − +	15	12	3	3375	2988	387	80.0	20.0	88.5	11.5	*S*49
*SS*21	+ − ● − ● + ●● ●● + ● − ● − +	16	8	0	4096	3008	0	50.0	0.0	73.4	0.0	——
⋮		⋮										
*SS*53	+ − − + − + + − − + + − + − − +	16	16	0	4096	4096	0	100.0	0.0	100.0	0.0	*S*10
*SS*54	+ − ●● − ●● + ● + ●● − ●● − +	17	8	0	4913	3128	0	47.1	0.0	63.7	0.0	——
⋮		⋮										
*SS*103	+ − − + − + + − ○ − + + − + − − +	17	16	1	4913	4912	1	94.1	5.9	100.0	0.0	*S*53
*SS*104	+ ● − − ●●●● + + ●●●● − − ● +	18	8	0	5832	3488	0	44.4	0.0	59.8	0.0	——
⋮		⋮										
*SS*213	+ − − + − + + − ○ ○ − + + − + − − +	18	16	2	5832	5824	8	88.9	11.1	99.9	0.1	*S*23
*SS*214	+ ● − − ●●● + ● ● ● + ●●● − − ● +	19	8	0	6859	4088	0	42.1	0.0	59.6	0.0	——
⋮		⋮										
*SS*333	+ − − + − + + − ○ ○ ○ − + + − + − − +	19	16	3	6859	6832	27	84.2	15.8	99.6	0.4	*S*59
*SS*334	+ − ●●● − ●● + ● ● + ●● − ●●● − +	20	8	0	8000	4448	0	40.0	0.0	55.6	0.0	——
⋮		⋮										
*SS*685	+ − − + − + + − ○○ ○○ − + + − + − − +	20	16	4	8000	7936	64	80.0	20.0	99.2	0.8	*S*48
*SS*686	+ ● − ●● − − ● + + ● + + ● − − ●● − ● +	21	12	0	9261	5052	0	57.1	0	54.6	0	——
⋮		⋮										
*SS*1029	+ ● − ●● − − ● + + ● + + ● − − ●● − ● +	21	16	5	9261	9136	125	76.2	23.8	98.7	1.3	*S*75

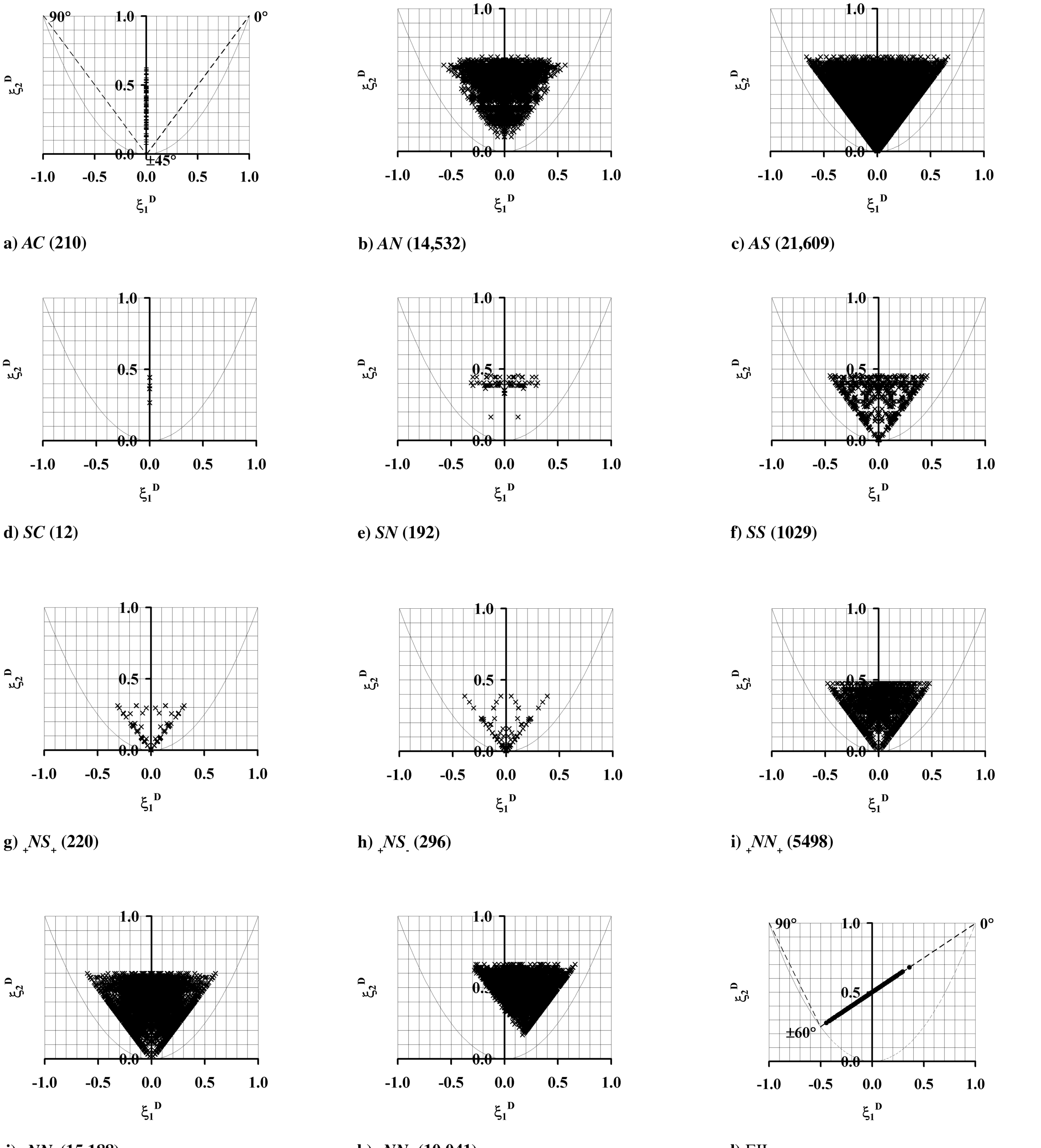

Fig. 1 Feasible domains of lamination parameters, in the form of Eqs. (25), for: a–k) FOLs, including form and number of sequences represented and; (l) the subset of $\pi/3$ EILs from the definitive list of FOLs.

0.1397 mm and stacking-sequence *NN*1071,

$$[+/-_2/\bigcirc_3/+_2/\bigcirc/-/+_2/-_3/\bigcirc_2/+]_T$$

the nondimensional parameters are verified by the calculations presented in Table 13, in which the first two columns provide the ply number and orientation, respectively. Subsequent columns illustrate the summations for each ply orientation of $(z_k - z_{k-1})$, $(z_k^2 - z_{k-1}^2)/2$, and $(z_k^3 - z_{k-1}^3)/3$, relating to the **A**, **B**, and **D** matrices, respectively. The distance from the laminate midplane, z, is expressed in term of ply thickness t, which is assumed to be of unit value.

The nondimensional parameters arising from the summations of Table 13 are $n_+ = {}_A\Sigma_+ = 6$, $n_- = 6$, $n_\bigcirc = 6$, $\zeta_+ = (4 \times {}_D\Sigma_+) = 1944$, $\zeta_- = 1944$, and $\zeta_\bigcirc = 1944$, where

$$n^3 = 18^3 = \zeta = \zeta_+ + \zeta_- + \zeta_\bigcirc = 5832$$

and $\zeta_\pm = 3888$. The **B** matrix summations confirm that $B_{ij} = 0$ for this laminate.

For fiber angles $\theta = \pm 45$ and 0 deg in place of symbols $\pm$ and $\bigcirc$, respectively, the transformed reduced stiffnesses are given in Table 14, which are readily calculated using Eqs. (17), and through Eqs. (15) and (16), the final stiffness matrices are derived for the laminate:

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ \text{sym} & & A_{66} \end{bmatrix} = \begin{bmatrix} 221,660 & 71,647 & 0 \\ & 94,956 & 0 \\ \text{sym} & & 73,661 \end{bmatrix} \text{N/mm}$$

$$\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ \text{sym} & & D_{66} \end{bmatrix} = \begin{bmatrix} 116,800 & 37,753 & 0 \\ & 50,035 & 0 \\ \text{sym} & & 38,814 \end{bmatrix} \text{N}\cdot\text{mm}$$

given that

$$A_{16} = \{n_\pm(n_+/n_\pm)Q'_{16+} + n_\pm(1 - n_+/n_\pm)Q'_{16-} + n_\bigcirc Q'_{16_\bigcirc} + n_\bullet Q'_{16_\bullet}\} \times t$$

$$A_{16} = A_{26} = \{12 \times (6/12) \times 37,791 + 12(1 - 6/12) \times -37,791 + 6 \times 0 + 0 \times 0\} \times 0.1397 = 0 \text{ N/mm}$$

$$D_{16} = \{\zeta_\pm/2 \times Q'_{16+} + \zeta_\pm/2 \times Q'_{16-} + \zeta_\bigcirc Q'_{16_\bigcirc} + \zeta_\bullet Q'_{16_\bullet}\} \times t^3/12$$

$$D_{16} = D_{26} = \{1944 \times 37,791 + 1944 \times -37,791 + 1944 \times 0 + 0 \times 0\} \times 0.1397^3/12 = 0 \text{ N}\cdot\text{mm}$$

Note that a modification to the fiber angles, with $\theta = \pm 60$ and 0 deg in place of ± 45 and 0 deg, respectively, leads to a FIL with

$$A_{11} = A_{22} = 173,473 \text{ N/mm} \qquad A_{12} = 56,482 \text{ N/mm}$$

$$A_{66} = (A_{11} - A_{12})/2 = 58,496 \text{ N/mm}$$

$$D_{11} = H^2/12 \times A_{11} = (n \times t)^2/12 \times A_{11} = D_{22} = 91,409 \text{ N}\cdot\text{mm}$$

$$D_{12} = 29,762 \text{ N}\cdot\text{mm} \qquad D_{66} = 30,823 \text{ N}\cdot\text{mm}$$

This modified laminate has the following lamination parameters (i.e., coordinates in Fig. 1), which are derived from Eq. (25):

$$\xi_1^D = \xi_9 = \{3888 \times \cos(2 \times 60\,\text{deg}) + 1944 \times \cos(2 \times 0\,\text{deg})\}/5832 = 0$$

$$\xi_2^D = \xi_{10} = \{3888 \times \cos^2(2 \times 60\,\text{deg}) + 1944 \times \cos^2(2 \times 0\,\text{deg})\}/5832 = 0.5$$

V. Fully Orthotropic Tapered Laminates

The following examples provide some insight into the scope for composite tailoring that the new definitive list provides. Two examples of tapered laminates are illustrated, both of which possess nonsymmetric stacking sequences. The first example involves single-ply terminations at the laminate midplane, and the second involves two-ply terminations on either side of the laminate midplane. Note that the two examples are similar if the sequences from the first example are taken to represent the symmetric half of the full laminate.

The following series provides the laminate number, together with the details of the particular ply to be terminated [i.e., ply orientation $(+/-/\bullet/\bigcirc)$] and corresponding ply number:

$$NN3979(\bullet_{11}) \to NN1796(\bullet_{10}) \to NN560(\bigcirc_{11}) \to NN153(\bigcirc_{10}) \to NN21(\bigcirc_9) \to NN2$$

Hence, terminating the 11th ply of sequence $NN3979$, corresponding to a ● ply, gives sequence $NN1796$. Terminating the 10th ply of sequence $NN1796$, which is also a ● ply, gives sequence

Table 13 Calculation procedure for the nondimensional parameters of the ABD relation

		A				B				D			
			${}_A\Sigma_\bigcirc$	${}_A\Sigma_-$	${}_A\Sigma_+$		${}_B\Sigma_\bigcirc$	${}_B\Sigma_-$	${}_B\Sigma_+$		${}_D\Sigma_\bigcirc$	${}_D\Sigma_-$	${}_D\Sigma_+$
Ply	θ	$(z_k - z_{k-1})$	6	6	6	$(z_k^2 - z_{k-1}^2)/2$	0	0	0	$(z_k^3 - z_{k-1}^3)/3$	486	486	486
1	+	1			→ 1	−17			→ −17	217			→ 217
2	−	1		→ 1		−15		→ 15		169		→ 169	
3	−	1		→ 1		−13		→ 13		127		→ 127	
4	○	1	→ 1			−11	→ −11			91	→ 91		
5	○	1	→ 1			−9	→ −9			61	→ 61		
6	○	1	→ 1			−7	→ −7			37	→ 37		
7	+	1			→ 1	−5			→ −5	19			→ 19
8	+	1			→ 1	−3			→ −3	7			→ 7
9	○	1	→ 1			−1	→ −1			1	→ 1		
10	−	1		→ 1		1		→ 1		1		→ 1	
11	+	1			→ 1	3			→ 3	7			→ 7
12	+	1			→ 1	5			→ 5	19			→ 19
13	−	1		→ 1		7		→ 7		37		→ 37	
14	−	1		→ 1		9		→ 9		61		→ 61	
15	−	1		→ 1		11		→ 11		91		→ 91	
16	○	1	→ 1			13	→ 13			127	→ 127		
17	○	1	→ 1			15	→ 15			169	→ 169		
18	+	1			→ 1	17			→ 17	217			→ 217

Table 14 Transformed reduced stiffness (N/mm^2) for IM7/8552 carbon-fiber/epoxy with $\theta = -45$, 45, 0, and 90 deg

θ	Q'_{11}	Q'_{12}	Q'_{16}	Q'_{22}	Q'_{26}	Q'_{66}
−45	50,894	40,554	−37,791	50,894	−37,791	41,355
45	50,894	40,554	37,791	50,894	37,791	41,355
0	162,660	4369	0	11,497	0	5170
90	11,497	4369	0	162,660	0	5170

*NN*560, and so on. The laminate sequences corresponding to this tapering series can be illustrated in full as follows:

NN 3979	+ − ● ○ ○ ● − + ○ ● ● ○ ○ − ● + + ● − ○
NN 1796	+ − ● ○ ○ ● − + ○ ● ○ ○ − ● + + ● − ○
NN 560	+ − ● ○ ○ ● − + ○ ○ ○ − ● + + ● − ○
NN 153	+ − ● ○ ○ ● − + ○ ○ − ● + + ● − ○
NN 21	+ − ● ○ ○ ● − + ○ − ● + + ● − ○
NN 2	+ − ● ○ ○ ● − + − ● + + ● − ○

In this second example, the series provides the laminate number, together with the details of each of the two plies to be terminated:

$$NN4104(\bigcirc_{6,16}) \to NN600(\bullet_{4,15}) \to NN33$$

Hence, terminating the 6th and 16th plies of sequence *NN*4104, both of which are ○ plies, gives sequence *NN*600. Terminating the 4th and 15th plies of sequence *NN*600, both of which are ● plies, gives sequence *NN*33. The laminate sequences corresponding to this tapering series can be illustrated in full as follows:

NN 4104	+ − ○ ● ○ ○ − + − + + − ○ − + ○ ● + − ○
NN 600	+ − ○ ● ○ − + − + + − ○ − + ● + − ○
NN 33	+ − ○ ○ − + − + + − ○ − + + − ○

A very different form of taper was demonstrated in [18], which is of relevance to the current paper. Here, a lightweight syntactic film core was used as a partial replacement for carbon-fiber material and for maintaining the fully orthotropic laminate properties. This was achieved by combining both symmetric and antisymmetric fully orthotropic angle-ply stacking sequences, which produced a FOL insensitive to the replacement of carbon-fiber plies with equivalent-thickness isotropic syntactic film core at the laminate midplane. This would not have been possible with nonsymmetric angle-ply subsequences, because the replacement of the angle plies by isotropic material, symmetrically displaced about the laminate midplane, would have caused an imbalance in the angle plies that remained, thus destroying the fully orthotropic nature of the laminate.

VI. Conclusions

The definitive list of fully orthotropic laminates (FOLs) with up to 21 plies has been presented in abridged form. The great majority of sequences have been shown to be of nonsymmetric form and many of these are without any subsequence patterns (e.g., symmetry or repeating groups), which is contrary to the assumptions on which many previous studies have been based.

The definitive list has been shown to contain all quasi-isotropic or extensionally isotropic laminates (EILs), fully isotropic laminates (FILs), and quasi-homogeneous orthotropic laminates (QHOLs), with up to 21 plies, which are subsets of FOLs. Finally, the scope for composite tailoring using FOLs has been demonstrated in the context of fully orthotropic tapered laminates.

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